

SOME RESULTS ON U-SHAPED LEARNING

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Abstract. *U-shaped learning behaviour* in cognitive development involves learning, un-learning and re-learning. It occurs, for example, in learning irregular verbs, which is a central topic in the cognitive science debate about learning models. The prior cognitive science literature is occupied with *how* humans do it. Instead, we are mostly concerned with whether U-shaped learning behaviour may be *necessary* in the abstract mathematical setting of inductive inference from positive data, that is, in Gold-style computational learning theory. Previous work ([1]) showed that U-shaped learning behaviour is *necessary* for *behaviourally correct learning*, i.e. when we require convergence in the limit to a possibly infinite sequence of correct conjectures, but not for syntactically convergent learning in the limit (= *explanatory learning*). In [2] we establish the necessity for the whole hierarchy of classes of *vacillatory learning*, where we require convergence in the limit to at most $k > 1$ grammars: every non U-shaped vacillatorily learnable class is already learnable in the explanatory sense (i.e. by converging in the limit to a single correct conjecture). Furthermore, if vacillatory learning with the parameter $k = 2$ is possible then non U-shaped *behaviourally correct learning* is also possible. But for $k \geq 3$, surprisingly, there is a class witnessing that this implication fails. In the sequel paper [3], we investigate two variants of non U-shaped learning: the first forbids returning to old *wrong* hypotheses while the second forbids return to old "overgeneralizing" conjectures (r.e. indexes enumerating a non-subset of the target language). We show, among other things, that (a) both variants restrict learning power of vacillatory learning, (b) return to wrong conjectures is more restrictive than return to correct conjectures and than return to overgeneralizing conjectures for explanatory learning, (c) return to wrong conjectures is necessary for full power of behaviorally correct machines, while (d) any class learnable by a behaviorally correct machine is so learnable by a machine that does never return to old overgeneralizing conjectures.

REFERENCES

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