

Esercizi sui sistemi lineari

1. Risolvere i sistemi lineari omogenei $\underline{u}' = A\underline{u}$ quando

$$A = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}, \quad A = \begin{pmatrix} 0 & -4 \\ 1 & 0 \end{pmatrix}, \quad A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}.$$

2. Risolvere i sistemi omogenei

$$\begin{cases} x' = x \\ y' = 3x - 2y \end{cases}, \quad \begin{cases} x' = -4y \\ y' = x \end{cases}, \quad \begin{cases} x' = 3x - 4y \\ y' = x - y \end{cases}.$$

3. Risolvere i sistemi omogenei

$$\begin{cases} x' = x + 3y - 2z \\ y' = 2y + 4z \\ z' = y + 2z \end{cases}, \quad \begin{cases} x' = -y + z \\ y' = -2x - y - 6z \\ z' = -y + z \end{cases}.$$

4. Risolvere il problema di Cauchy $\underline{u}' = A\underline{u}$, $\underline{u}(0) = \underline{x}$ quando

$$A = \begin{pmatrix} 4 & -3 \\ 8 & -6 \end{pmatrix}, \quad \underline{x} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}; \quad A = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 1 & -4 \\ -1 & -1 & 4 \end{pmatrix}, \quad \underline{x} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}.$$

5. Risolvere il sistema $\underline{u}' = A\underline{u} + \underline{f}$ quando

$$A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}, \quad \underline{f}(t) = \begin{pmatrix} et \\ e2t \end{pmatrix}; \quad A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \quad \underline{f}(t) = \begin{pmatrix} t \\ 1 \\ 0 \end{pmatrix}.$$

6. Risolvere i problemi di Cauchy

$$\begin{cases} x' = -y + e^{-t} \\ y' = x + \sin t \\ x(0) = 1, y(0) = 0 \end{cases}, \quad \begin{cases} x' = -3x + t^2 \\ y' = 3y - z + t \\ z' = y + z \\ x(0) = 1, y(0) = -1, z(0) = 0 \end{cases}.$$