Mathematical Memories

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My first memory related to Mathematics: Sicily, summer of 1971. I was nine years old, and we were spending a holiday in Marzamemi, a village in the southeastern tip of the island, close to Siracusa. Though we lived in northern Italy, my father's family was from Catania, on the east coast of Sicily, about one hundred kilometers north of Marzamemi, and so this was a reasonable compromise between visiting relatives and preserving our own space (I love my Sicilian relatives, but sometimes they could be overwhelming—or at least that was my impression at the time. Now it feels different; getting older my perspective has changed, and my relatives have too.)

We rented an apartment having a large terrace with a sea view—or at least I think it had a sea view: I don't really remember that. What I do remember very clearly are the bright Sicilian light, the sky so deeply blue that a polarizing filter would render it black against sparkly white puffy clouds, the square-tiled floor of the terrace and—most of all—my father explaining Pythagoras' theorem to me by making use of the square tiles and a rough wooden stick.

Now let me pause a second to explain the context in which this pivotal moment in my life was happening. My father was not a scientist, leave alone a mathematician; nor had he had any training in science. The closest he got to science was enrolling (under family pressure) in an engineering degree at the university of Catania; but he dropped out almost immediately, leaving Sicily for Milan in northern Italy, to pursue his dreams of becoming a journalist—and yes, also to pursue a girl. It worked; he married the girl (and a few years later they had two sons, the first one being me), he became quite a good journalist, and he had a successful, daring, and innovative career in the Italian editorial industry. He also was a good watercolors painter, something he inherited from his father and his uncle (both professional painters), and he liked to write and illustrate deliciously silly stories for his sons (and later his grandsons); but mathematics was just not his thing. My mother? My mother is a very intelligent woman, extremely well organized, with a lot of different interests, but again no scientific training. She went to work as a secretary immediately after completing high school (she would have liked to enroll at the university in a chemistry degree, but her family could not afford it), and kept working until my father started earning enough to support both of them—actually, the three of us, because this coincided (not surprisingly: as I've already said, my mother really is quite good in organizing things) with my birth. Relatives, then? As long as anyone remembers, nobody in any of both my father's and my mother's families has ever shown any interest or inclination for mathematics (and this is still true as of today; maybe one of my sons ... but that might be wishful thinking on my part). So everybody was a bit flummoxed by having around this little child clearly loving numbers and playing with them; and most of my relatives didn't know what to do with me. (Actually, they might have had some reasons. I was a bright, happy, healthy, social kid who loved taking out from the shelves his starting-to-become-impressive comic books collection just to have the pleasure of putting it back in place in the right order; or

that spent time reading Tito Livio's *History of Rome* to reenact the main battles with toy soldiers . . .)

My parents loved me, and did all in their power to support my inclinations, and foster new ones. Beside lending me his copies of Tito Livio's, or of Dante Alighieri's *Divina Commedia,* (or of Shakespeare's tragedies, for that matter), my father organized wonderful journalism classes for me and my friends; and my mother encouraged and endured my chemistry and culinary experiments in the kitchen (the fine line between them sometimes crossed with frankly embarrassing results). Anyway, I really have no idea why and how he decided to do it, but that Sicilian morning my father clearly liked the possibility of explaining to me something mathematical that he understood, and he used all his rhetorical abilities to present me this surprising (and fascinating) fact of mathematical life in the sun-drenched square-tiled magical terrace in southern Italy.

The next thing I remember is convincing him to raid a local stationery store, small, dark, dusty, where a perplexed clerk was able to recover, in a far away corner in a back room, buried under piles of abandoned papery things too forgotten to be of any use, a geometry middle school textbook. I still clearly recall its pink-yellow cover, with *Matematica* written in capital white letters . . . I read and re-read it several times that summer. I cannot say now whether it was any good from a standard didactical point of view; but for me it worked perfectly as a guide into the discovery of an exciting new world. It was the best gift my parents could have given me (that year: they did get me wonderful gifts in the subsequent years too. I once asked for a Tangram book; my father got back with an Origami book instead—all Japanese pastimes, right?—starting a love affair still going strong today.)

Let's move forward a couple of years: Inzago (my home town, a small village nearby Milan in northern Italy), winter of 1973. In bed one night, being still too young for thinking about girls, I thought about numbers . . . and I made a remarkable discovery: summing consecutive odd numbers one gets square numbers! More precisely, I realized that

$$1 + 3 = 4 = 2^2 \ ,$$
$$1 + 3 + 5 = 4 + 5 = 9 = 3^2 \ ,$$
$$1 + 3 + 5 + 7 = 9 + 7 = 16 = 4^2 \ ,$$
$$1 + 3 + 5 + 7 + 9 = 16 + 9 = 25 = 5^2 \ldots$$

the pattern was too clear to be a mere coincidence! The next day I told my discovery to an engineering student who was visiting my parents' house; my father, having successfully emigrated from far away poor Sicily to the rich mythical Milan, economical capital of Italy, liked whenever possible to welcome and help people just arrived from Catania. I don't remember the student's name, possibly Michele, I'm not sure. I remember his small, bespectacled, intelligent face; for some reason he liked to talk to me, and I responded to his interest in me—slightly flattered by his attention, even though there was something in him that somehow didn't fully convince me, like those potentially good cakes just a bit too sweet. He and his friend (blond and athletic, as much as he was dark and wiry, at least in my memory) were frequent guests of my parents and their friends for a while; and then disappeared. I have no idea what became of them.

Anyway, he was visiting my parents' house the day after my "discovery", and so I asked him whether he knew that and whether it was always true. He smiled, took out a pen and a piece of paper, and answered; "But yes of course, what you found out is this

$$(1) \qquad \sum_{j=0}^{n}(2j+1) = (n+1)^2 \, ,$$

see? It's clear!". I was a bit perplexed, even after he explained the meaning of the summation symbol. To me, this seemed just a restatement of what I said, and not an explanation of why it was always true. Then and there I decided that engineering was not for me. I mean, engineers are wonderful people and we cannot do without them, but they are not scientists—or, at least, not mathematicians. As a budding mathematician, I already was much more interested in what was true and *why* it was true, whereas for engineers ... "Who cares whether it is true or not, let's see what we can do with it!". OK, maybe this is exaggerating a bit, but still ...

[The proof of (1) that now I like the most is geometrical. Take a square of side n, made up by n^2 unit squares. To build a square of side $n+1$ one draws a row of n unit squares above, a column of n unit squares on the right, and a single unit square in the upper-right corner. In other words, to go from n^2 to $(n+1)^2$ one needs to add exactly $2n+1$, the next odd number; so starting from 1 and adding up odd numbers one recovers all square numbers, as I found out that night. Notice that this argument can also be expressed by the algebraic formula $(n+1)^2 = n^2 + (2n+1)$, which is the binomial expansion of the square, giving a nice connection between geometry and algebra.]

Before going on to my teenager years, I should also mention that, toward the end of middle school, I ran into (the Italian edition of) Martin Gardner's books, and I immediately became addicted to them. I think I read those books thousands of times, and I still pick them up from time to time. But since this is an experience shared by most (if not all) mathematicians in my age range, I'll limit myself to a single suggestion: go to the closest bookshop and buy one of them, any one. If you are reading this you will love it, I promise, and you'll beg for more.

Being a teenager in the second half of the Seventies in Italy was ... let's say interesting, as in the famous Chinese curse. The dreams of the Sixties were fast becoming nightmares, poisoned by terrorism, coming both from the extreme right (the most lethal, with several bombs and hundred of people killed) and from the extreme left (with a much smaller body count, just a few units, but much more pervasive among young people and much more disruptive of the love, peace and harmony dreams of the Sixties). I enrolled in an experimental high school, one of the few then offering a curriculum in computer science—at the time (can you believe it?) Italy was at the forefront of world research in computers, both for software and hardware, a position idiotically given up in the Eighties by reconverting or closing down the major hardware firms and diverting most funds outside of research. As a country, we are still paying for the incredibly bad political and economical decisions that were made in that decade.

Mathematics was still one of my main interests, but not the only one. I loved programming; I wrote a few videogames (in Assembler! Memory was a precious and scarce commodity, and you had to make good use of every single byte), also devising some primitive-

but-not-so-inefficient artificial intelligence algorithms based on Shannon's entropy. At the time video screens were mostly black and green, only able to display characters; but since e-mails hadn't started yet, @ made for perfectly scary monsters.

For a couple of years I also got somewhat involved in politics; somebody even predicted for me a future as a prominent member of some leftist party . . . Luckily, that prophecy did not come to pass; but the experiences I made then are turning out to be quite useful now that I'm immersed up to my neck (and sometimes even more) in academical politics.

The artistic side of my family's legacy started to become more evident. I took up singing, and I learned how to play rhythm guitar to accompany myself. I also started having photography as a serious hobby (again because of a gift of my father: his old camera, totally manual, no autofocus, no exposure meter, a wonderfully effective teaching tool—and a fast glimpse to my web site shows how important photography still is for me today). But mostly, I started writing in earnest, mainly short stories, slowly finding my own voice (and bringing me to become several years later, for some time, a professional comic book writer. It is sort of sobering to realize that in Italy the number of people having read one of my comic book stories is way higher than the number of people having read one of my mathematical papers, even though the number of my published mathematical papers is ten times the number of my published comic book stories . . .). I also wrote some literary criticism; in particular, with a group of (older) friends we started a science-fiction fanzine that in a couple of years became much more, mixing structuralism, semiotics, cutting-edge philosophy, politics, happenings (funny, odd and unexpected, having then quite an impact on the cultural milieu in Milan, and still remembered today) with plain undeterred love for good stories.

And yes, there were girls too. I fell in love with some, and some fell in love with me; and when both things happened at the same time, well . . . let's just say that teenagers have a knack of making a mess of first loves, and of getting hurt in the process, and let's leave it at that.

But mathematics was still there. I borrowed from the school library (and, to my shame, I must admit I never returned it; but nobody else was reading it . . . and when I realized that's bollocks it was years too late to make amends) the Italian edition of Courant-Robbins' *What Is Mathematics?*, and not surprisingly I just loved it; I think I read it at least three times. Most of the mathematics professors in my school weren't that good (on the other hand, I had an excellent teacher of Italian literature); but there was one that I liked, and I remember going to him asking: "Ok, now I know all of algebra, geometry, trigonometry, and calculus; what else is there?". I still see him smiling while answering "Well, maybe 'all' is too strong a word . . . "

A turning point in my mathematical training was picking up the book *Istituzioni di Algebra Astratta* by Lucio Lombardo Radice (the title means *Abstract Algebra Institutions,* where in Italy "Institutions" is used to denote courses containing the most important parts of a given subject), then in its twelfth edition. And a whole world opened to me . . . not because of that book in particular (it wasn't that good), but because it was just one in a whole series of mathematical books, including for instance the Italian edition of P.J. Cohen's *Set Theory and the Continuum Hypothesis* (which fascinated me even though I didn't really understand it at the time). And looking for them brought me in sections of

bookstores I never went before, where I found many more books ... I recall very clearly the huge basement floor of Hoepli bookstore in downtown Milan (Hoepli being a very old publisher, specialized in publishing technical manuals in all branches of science and technology), where I found for instance the three volumes of Don Knuth's *The Art of Computer Programming*, that I still devotedly cherish today (and even used for one of my mathematical papers last year); or F. Harary's *Graph Theory,* that sparked my interest in, you guessed it, graph theory, up to the point that I wrote what I might rightfully consider my first mathematical paper, proving with an elaborate construction a theorem about Hamiltonian graphs. (A few years later I checked and found out that "my theorem" was of course already known, but with a different—though equivalent—formulation and with quite a different proof; I wonder whether something else could be extracted from my method of proof ... I should still have the manuscript somewhere ...) I also remember stating proudly that I would devote myself to "modern, contemporary mathematics," that is, discrete mathematics, nothing of that continuous, old, stuffy, analytic sort of things for me, oh no ... Well ... I must admit that I did not really follow up my bold statement, and I've instead become an expert in complex analysis, a branch of mathematics so classical as to be considered the queen of nineteenth century mathematics ... but maybe this is why I now at least prefer discrete dynamics to continuous dynamics ...

At last high school ended, and I had to decide what to do at the university. I could choose whatever I wanted: in Italy then with a (five years) high school degree one could enroll in almost all university undergraduate courses, with no limitations; it was enough to pay the (not exceedingly high) university fees. The only exceptions were a very few high-level institutions geared toward the best students, like the Scuola Normale Superiore of Pisa, in central Italy; to be admitted there it was necessary to pass a hard entrance examination. Of course, the entrance examination depended on the chosen major; in particular, in Science one could choose between Mathematics, Physics, Computer Sciences, Biology, and Chemistry, while in Humanities the choices were History, Philosophy, Art History, Italian, and Classical literature. I was not really convinced that I could be admitted there; but it was worth a try, and at worst I just would had had a good excuse for a trip to Pisa. But this meant I definitely had to decide a major (also because to enter the Scuola Normale Superiore one also has to enroll in the University of Pisa and attend the corresponding courses there; the symbiosis between Scuola Normale and the University of Pisa runs very deep, none of them could survive without the other).

In my last year of high school I devoted a lot of time to reading (and writing) literary criticism, philosophy, semiotics and the like; so I might have gone for a literary major, and nobody would have been surprised. But in my mind it was clear that I was better (and better prepared: I studied Latin only one year in middle school, and forgot all of it) as a mathematics or computer science student than as a humanities student, so that was settled. But the choice between mathematics and computer science ... that was much harder. I decided only when I was in line for presenting my application for admission to the Scuola Normale Superiore, and even there I waited as long as possible. I'd left blank the space where I should have put my major; and only at the very last minute, when it was my turn to turn in the application, I hastily jotted down "Matematica" on the dotted line. A couple of months later I passed the entrance examination (that year only 12 students

with a Science major were admitted: 8 physicists, 1 biologist, and 3 mathematicians); and so in November 1981 I moved to Pisa, to start my life as a mathematician.

I do not regret at all that choice; I do regret other choices I did at that time (for instance, that day with that girl I should have . . . never mind), but choosing mathematics was and has been the right thing for me. Mathematics in my experience mostly is a way of thinking. I can feel my brain working in different ways when I'm doing mathematics (up to the point that my mother is able to understand when I'm working by the way I answer her telephone calls) with respect to when I'm writing stories, or taking pictures, or singing, or folding origami, or dealing with academical politics and bureaucracy, and even if I can do all those other things (and a few more, some even including women) in a reasonably competent manner, thinking in a mathematical way feels the most natural to me.

Looking back now I realize how much my life has been influenced by chance encounters or events; I wonder what would have happened if that dusty stationery store had no middle school mathematics book stashed away in that dark back room . . . so let me end with another example of a chain of apparently unrelated events having lasting consequences. When I was a Ph.D. student, my girlfriend got an exchange fellowship with the University of California, Berkeley (she actually got it before becoming my girlfriend, but because of another complicated sequence of very happy coincidences she was able to go there one year later). That brought me to pester my Ph.D. advisor (Edoardo Vesentini, to whom I'm incredibly grateful for putting up with my sometimes weird requests—and because he has been a great inspiration to me during all these years) for finding me too a way to go to Berkeley, which he did. In Berkeley I met and befriended Steve Krantz, who was there just for a couple of days giving a talk. And that eventually led, twenty-five years later, to me writing this very piece you are finishing reading just now.

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